

The Complete character table for the point group C_{2v} is as follows -

C_{2v}	E	$C_2(z)$	σ_{xz}	σ_{yz}		
A_1	1	1	1	1	Z	x^2, y^2, z^2
A_2	1	1	-1	-1	R_z	xy
B_1	1	-1	1	-1	x, R_y	xz
B_2	1	-1	-1	1	y, R_x	yz
I	II				III	IV

In the upper left corner is the Schoenflies notation for the group and the upper row of the table consists of the symmetry operators grouped into classes.

Area I: Area I represents the symbol for irreducible representation according to Mulliken.

All these irreducible representations are unidimensional and hence A or B symbol has been used.

Because upper two irreducible representations are symmetrical with respect to the principal axis and hence A's are written and lower two representations are anti-symmetrical with respect to the principal axis and hence B's are written.

Subscript 1 or 2 are written for the symmetric/and antisymmetric respectively with respect to σ_{xz} .

Area II: Area II of the character table has the characters of the irreducible representations of the point group C_{2v} .

Area III: Area III represents the Cartesian Co-ordinates of rotational axis corresponding to irreducible representation. In order to assign the Cartesian Co-ordinates of rotational axis, we must perform the operations on them and enquire the characters.

Let us consider a vector along Z-axis. The operations E , $C_2(z)$, σ_{xz} , σ_{yz} do not change the direction of the head of vector. Hence its characters are 1, 1, 1, 1.

Thus the vector Z transforms under the symmetry operations into A_1 . Similarly x and y vectors transform into B_1 and B_2 representations respectively. Rotation axis R_x , R_y and R_z represents rotation about x, y and z axis.

To understand the transformation by rotational axis, we should mark a Curved arrow and Symmetry operation. If the direction of the head of arrow does not change operation, the character is +1 and -1 for the change of head of arrow.

	E	C ₂	σ _{xz}	σ _{yz}	Transformation
 R _z	1	1	1	-1	A ₂
 R _y	1	-1	1	-1	B ₁
 R _x	1	1	-1	-1	B ₂

Area IV: - Area IV represents Square and binary products to the irreducible representation. For assignment of the square and binary of vectors, the characters are squared or direct products are obtained, x^2 , y^2 and z^2 belongs to A₁ irreducible representation.

The product of character of x and y belongs to A₂, the product of x and z belongs to B₁ product of y and z belongs to B₂ representation.

Construction of Character Table for C_{3v} Point Group

There are six symmetry operations present in C_{3v} point group i.e. $E, C_3^1, C_3^2, \sigma_a, \sigma_b, \sigma_c$.

These operations are divided into three classes i.e. ($E, 2C_3, 3\sigma_v$) and hence there are three irreducible representations.

Let them be $\Gamma_1, \Gamma_2, \Gamma_3$.

The sum of the squares of dimensions (i.e. character of the identity operation) should be equal to six (or 6).

$$\sum l_i^2 = l_1^2 + l_2^2 + l_3^2 = 6$$

The values of l_i that will satisfy this requirement are 1, 1, and 2.

C_{3v}	E	$2C_3$	$3\sigma_v$
Γ_1	1		
Γ_2	1		
Γ_3	2		

Every point group possesses one representation which is totally symmetric.

In this representations, all the operations have the character value one (1).

Thus we have

C_{3v}	E	$2C_3$	$3\sigma_v$
Γ_1	1	1	1

It can be seen that the summation of the square of the character of the operations is equal to 6

$$1^2 + 2 \times 1^2 + 3 \times 1^2 = 6$$

Γ_2 (Second irreducible representation), must be orthogonal to Γ_1 . Since $\chi_2(E)$ must always be positive and hence Γ_2 must consist of three +1 and three -1. This is only possible if Γ_2 has 1, 1, and -1.

C_{3v}	E	$2C_3$	$3\sigma_v$
Γ_1	1	1	1

One third representations will be of two dimensions and hence $\chi_3(E)$ is 2.

In order to find out the values of $\chi_3(C_3)$ and $\chi_3(\sigma_v)$, we make the use of the orthogonality relationship.

Γ_1 and Γ_3 are orthogonal to each other and Γ_2 and Γ_3 are also

C_{3v}	E	$2C_3$	$3\sigma_v$
Γ_1	1	1	1
Γ_2	1	1	-1
Γ_3	2	$\chi_3(C_3)$	$\chi_3(\sigma_v)$

$$\sum_R \chi_1(R) \cdot \chi_3(R) = (1) \cdot (2) + 2(1) \cdot \chi_3(C_3) + 3 \cdot (1) \chi_3(\sigma_v) = 0 \quad \text{--- (i)}$$

$$\sum_R \chi_2(R) \cdot \chi_3(R) = (1) \cdot (2) + 2(1) \cdot \chi_3(C_3) + 3 \cdot (-1) \chi_3(\sigma_v) = 0 \quad \text{--- (ii)}$$

Solving equation (i) and (ii) we get

$$2\chi_3(C_3) + 3\chi_3(\sigma_v) = -2 \quad \text{--- (iii)}$$

$$2\chi_3(C_3) - 3\chi_3(\sigma_v) = -2 \quad \text{--- (iv)}$$

On adding eqⁿ (iii) and (iv) we get

$$4\chi_3(C_3) = -4$$

$$\therefore \chi_3(C_3) = -1$$

This value is substituted in equation (iii) to get

$$2(-1) + 3\chi_3(\sigma_v) = -2$$

$$3\chi_3(\sigma_v) = 0$$

$$\chi_3(\sigma_v) = 0$$

Thus, the complete set of characters of irreducible representations of C_{3v} point group is:

C_{3v}	E	$2C_3$	$3\sigma_v$
Γ_1	1	1	1
Γ_2	1	1	-1
Γ_3	2	-1	0